# Investigating Connections Between College-Level Statistics Courses and Teaching K-12 Mathematics

Rachel Tremaine & Elizabeth Arnold
*Colorado State University*

# META Math Project

The **M**athematical **E**ducation of **T**eachers as an **A**pplication of Undergraduate **Math**ematics (META Math) is a project to create and field-test annotated lesson plans for use in undergraduate statistics and mathematics courses that highlight connections to teaching school mathematics.

**Goal:** Increase awareness of these connections among all students in the course, even those not intending to pursue teaching as a career, in a way that deepens their understanding of undergraduate mathematics.

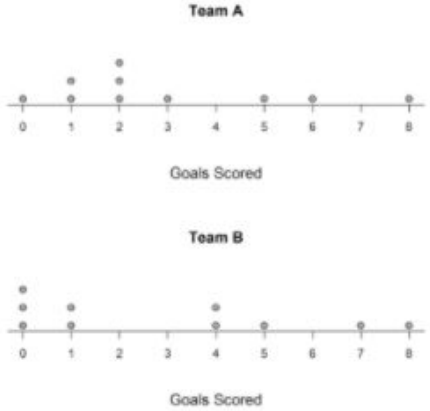
# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.
2. **Explaining Mathematical Content** – Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.
3. **Looking Back / Looking Forward** – Undergraduates can explain how mathematics topics are related over a span of K-12 curriculum through undergraduate mathematics.
4. **School Student Thinking** – Undergraduates evaluate the mathematics underlying a student's work and explain what that student may understand.
5. **Guiding School Students' Understanding** – Undergraduates pose or evaluate guiding questions to help a hypothetical student understand a mathematical concept and explain how the questions may guide the student's learning.

# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.

1. Two soccer teams will be meeting in the city championship game. Each team played 10 games and averaged 3 goals scored per game for the season. The two dotplots below show the number of goals scored by each team per game for the season.




a. Calculate the mean absolute deviation (MAD) for Team A.

# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.

2. **Explaining Mathematical Content** – Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.

> b. 95% of the sample mean body temperatures fall between _____ and _____. Explain how you came up with these endpoints.

# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.

2. **Explaining Mathematical Content** – Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.

3. **Looking Back / Looking Forward** – Undergraduates can explain how mathematics topics are related over a span of K-12 curriculum through undergraduate mathematics.

> 2. Consider different measures of variability.
>    a. Why might it be helpful for students to learn MAD (mean absolute deviation) before SD (standard deviation)?

# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.

2. **Explaining Mathematical Content** – Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.

3. **Looking Back / Looking Forward** – Undergraduates can explain how mathematics topics are related over a span of K-12 curriculum through undergraduate mathematics.

4. **School Student Thinking** – Undergraduates evaluate the mathematics underlying a student's work and explain what that student may understand.

> b. For **each** of the other three (incorrect) choices, describe what a student who selected that choice does and does not understand.

# 5 Types of Connections to Teaching (Arnold et al., 2020)

1. **Content Knowledge** – Undergraduates use course content in applied teaching contexts or to answer mathematical questions in the course.

2. **Explaining Mathematical Content** – Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.

3. **Looking Back / Looking Forward** – Undergraduates can explain how mathematics topics are related over a span of K-12 curriculum through undergraduate mathematics.

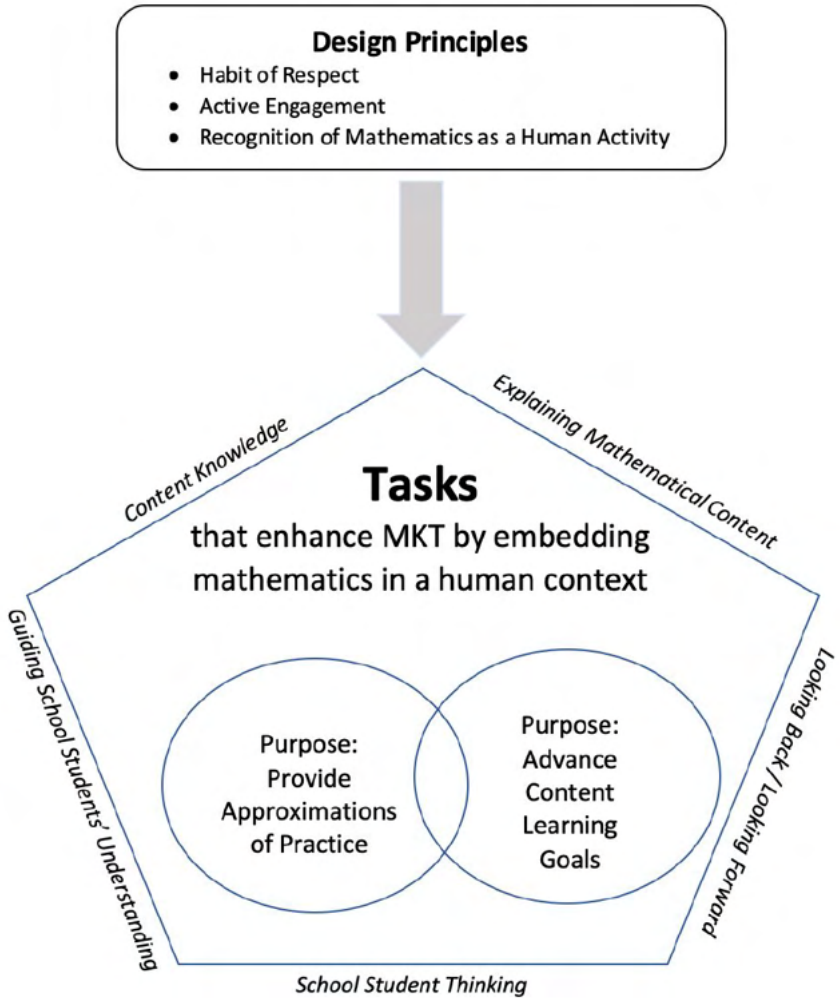
4. **School Student Thinking** – Undergraduates evaluate the mathematics underlying a student's work and explain what that student may understand.

5. **Guiding School Students' Understanding** – Undergraduates pose or evaluate guiding questions to help a hypothetical student understand a mathematical concept and explain how the questions may guide the student's learning.

> Write two questions you can ask Ariel to help him correct his work. Explain how your questions might help guide Ariel's statistical understanding.

# Design Principles for Creating These Tasks(Álvarez et al., 2020)

1. **Choose content** in the undergraduate course that is relevant for future teachers to understand at a deep level
2. Make **explicit connections** between the undergraduate mathematics/statistics and school mathematics



# Setting & Methodology

- Tasks emphasizing connections to teaching were embedded into two lessons: (1) ***Measures of Variability*** & (2) ***Margin of Error***
- **Implemented** in two undergraduate *Intro to Statistics* courses (Fall 2019)
  - Large Public University in the South
  - Small Private Liberal Arts College in the Midwest
- **41 undergraduates** consented to participate, 15 of which were interviewed (4 of which were pre-service teachers)
  - Interviews, exit tickets, assessments

*What is the nature of undergraduates' experiences with connections between college-level statistics and teaching K-12 statistics?*

(*Measures of Variability* & *Margin of Error*)

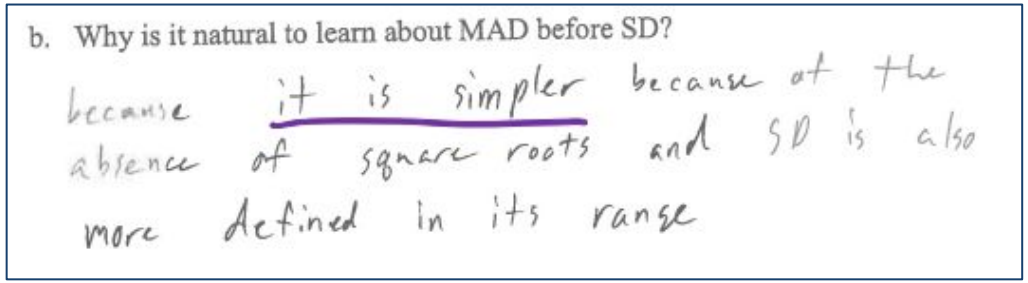
# A Few Key Findings

- Undergraduates primarily described their experiences (immediately post lesson) with *Content Knowledge* and *Looking Back / Looking Forward* connections in their exit tickets.
  - **CK:** [The *Variability* lesson] actually did help me, discussing it in class, because shortly afterwards I had some students bring in some MAD [mean absolute deviation] stuff and I was like, 'oh, how about that, I just covered this. This is perfect.' So [the lesson] definitely made things a little bit easier and more clear. *(Emily, PST)*
  - **LB/LF:** Instead of throw[ing] out all your math from high school… you can build on it and reinforce [it]. *(Belle)*

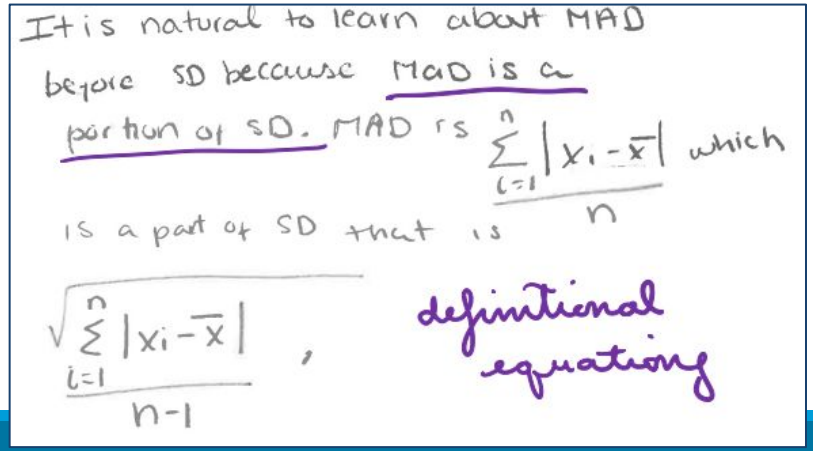
- Some undergraduates stated they experienced no connections or what they were learning was only applicable to AP Statistics in high school.
  - I think [the content in the *Margin of Error* lesson] would be useful especially in [an] AP Statistics class. I don't think for your typical algebra, geometry, algebra 2, break out kind of classes -- I don't think they would go in depth on margin of error so much. *(Emily, PST)*

# A Few Key Findings

- *Area for Improvement:* We can provide undergraduates more support to understand the topics now being taught in high school along with the progression of topics taught (i.e., a *Looking Back / Looking Forward* connection)
  - *About 70% of undergraduates indicated that MAD is simpler than SD in their responses to the following assessment question.*


b. Why is it natural to learn about MAD before SD?

because it is simpler because of the absence of square roots and SD is also more defined in its range


It is natural to learn about MAD before SD because MAD is a portion of SD. MAD is $\frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$ which is a part of SD that is $\frac{\sqrt{\sum_{i=1}^{n} |x_i - \bar{x}|}}{n-1}$, definitional equation

# A Few Key Findings

- Undergrads described the novelty, value, and humanity in School Student Thinking connections. These tasks: (1) deepened their own understanding, (2) exposed them to different perspectives and approaches, (3) encouraged internal reflections, (4) offered opportunities to find value in hypothetical student work (habit of respect)
  - ***Josie:*** SST tasks emphasized that "everyone understands things differently"
  - ***Raymond:*** "So sometimes I can explain why I think something that way but seeing somebody else's thinking and having to make a judgment of whether that's right or wrong was something that was different. That's something I haven't seen tested on before and I thought that was very interesting. So it … did force me to think about why I didn't agree with these [choices]. And I think that was good for me."
  - ***Grace:*** SST tasks "help[ed] me recognize, 'oh the students make certain mistakes but they also do things correctly too.'"

# A Few Key Findings

- *Area for Improvement:* Undergraduates tended to gravitate toward emphasizing what a hypothetical student *did not* understand as opposed to what she *did* understand

> b. For **each** of the other three (incorrect) choices, describe what a student who selected that choice does and does not understand.

A - May not understand how a larger sample size affects a distribution, it is very similar to Ben's dotplot.

C - If pulling from the same data set, her mean wouldn't be completely different

D - Again, A larger sample size wouldn't make a dotplot more "spread out"

# A Few Key Findings

- Statistical content held on par with connections to teaching
  - Assessment responses were where one would expect given content
  - ***Ann:*** lessons “didn’t disrupt [the undergraduates]”
  - ***Emily:*** lesson “doesn’t hurt” student learning

# A Few Tips for Developing Tasks that Highlight Connections to Teaching

- Choose a topic that is **foundational** to school mathematics
- **Include human beings** as characters in tasks
  - Makes space for *Habit of Respect* principles
  - Humanizing of Mathematics (Gutiérrez, 2018)
    - "The Role of Human Beings as Characters in Tasks for Content Courses" (Arnold, Fulton, Tremaine [in preparation!])
- Provide opportunities for undergraduates to share an **anti-deficit perspective** about hypothetical student work
  - Pre-service teachers need to understand how to encourage, guide, and emphasize what hypothetical students do *well,* not just how to critique or correct mistakes. Prompt undergraduates to explain what a student does and does not understand in separate questions.

# Acknowledgements

**PIs:** James Alvarez, Elizabeth Burroughs, Doug Ensley, Nancy Neudauer, James Tanton

**Researcher Group:** Elizabeth Arnold, Elizabeth Fulton, Andrew Kercher

**Graduate Research Assistants:** Kyle Turner, Rachel Tremaine

**Funding:** The project is funded by NSF DUE-1726624.

*To request access to our lessons, please visit:* [http://tinyurl.com/METAMathLessons](http://tinyurl.com/METAMathLessons)

MAA MATHEMATICAL ASSOCIATION OF AMERICA

MONTANA STATE UNIVERSITY

UNIVERSITY OF TEXAS ARLINGTON

Colorado State University

NSF

# References


- Álvarez, J. A. M., Arnold, E. G., Burroughs, E. A., Fulton, E. W., & Kercher, A. (2020). The design of tasks that address applications to teaching secondary mathematics for use in undergraduate mathematics courses. *Journal of Mathematical Behavior. 60*: 100814.

- Arnold, E. G., Burroughs, E. A., Fulton, E. W., & Álvarez, J. A. M. (2020). Applications of Teaching Secondary Mathematics in Undergraduate Mathematics Courses. *TSG33 of the 14th International Congress on Mathematical Education*. Shanghai, China: International Mathematical Union. [Paper delayed until July 2021 due to COVID-19 pandemic]. Available at arXiv:2102.04537

- Gutiérrez (2018). The need to Rehumanize Mathematics. In I. Goffeney & R. Gutierrez (Eds). Rehumanizing Mathematics for Black, Indigenous, and Latinx Students (pp. 1-10). Reston, VA: National Council of Teachers of Mathematics.