

**The Central Limit Theorem: A Group Activity to Die For!**  
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**STA-282 Group Activity: The Central Limit Theorem**

Let  $X$  be a random variable representing the roll of a fair 6-sided die. Complete the following table which will represent the **theoretical** distribution of  $X$  (Value of the Die and it's corresponding probability).

Table 1

Die Value ( $x$ )	Probability that $X = x$

1. Now we will conduct our experiment:
  - a. Roll your die 4 times and calculate the average of these 4 rolls ( $\bar{x}$ ) in the first box (Roll #1) in the table provided below.
  - b. Now repeat step “a” 40 times. You have 40  $\bar{x}$  in Table 2

Table 2

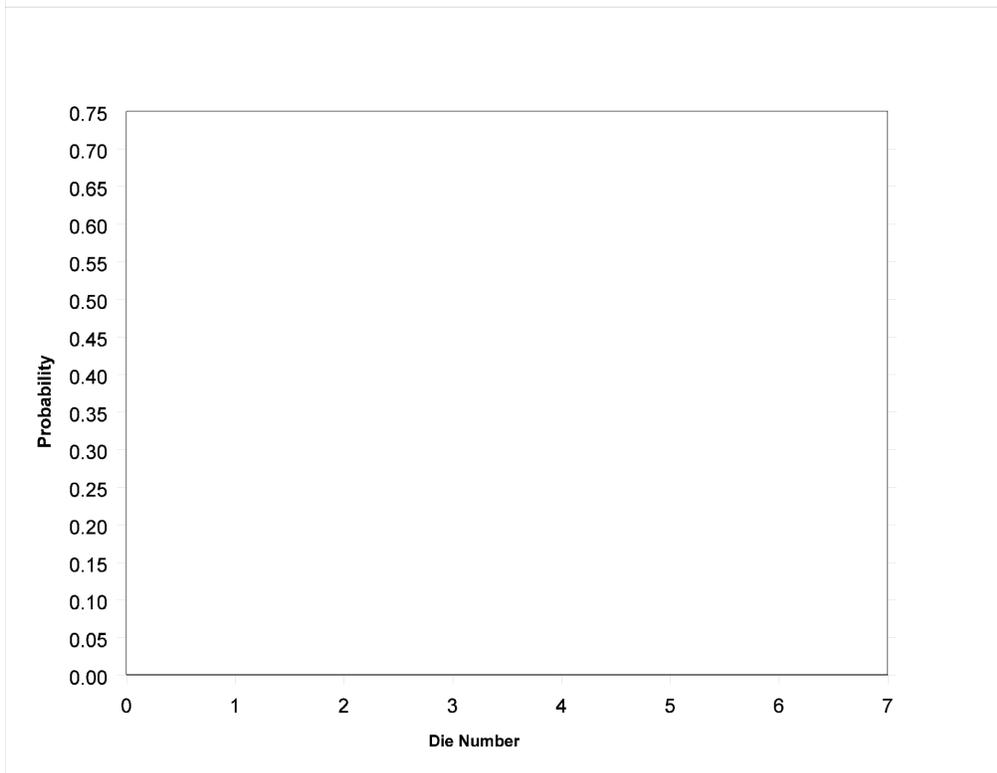
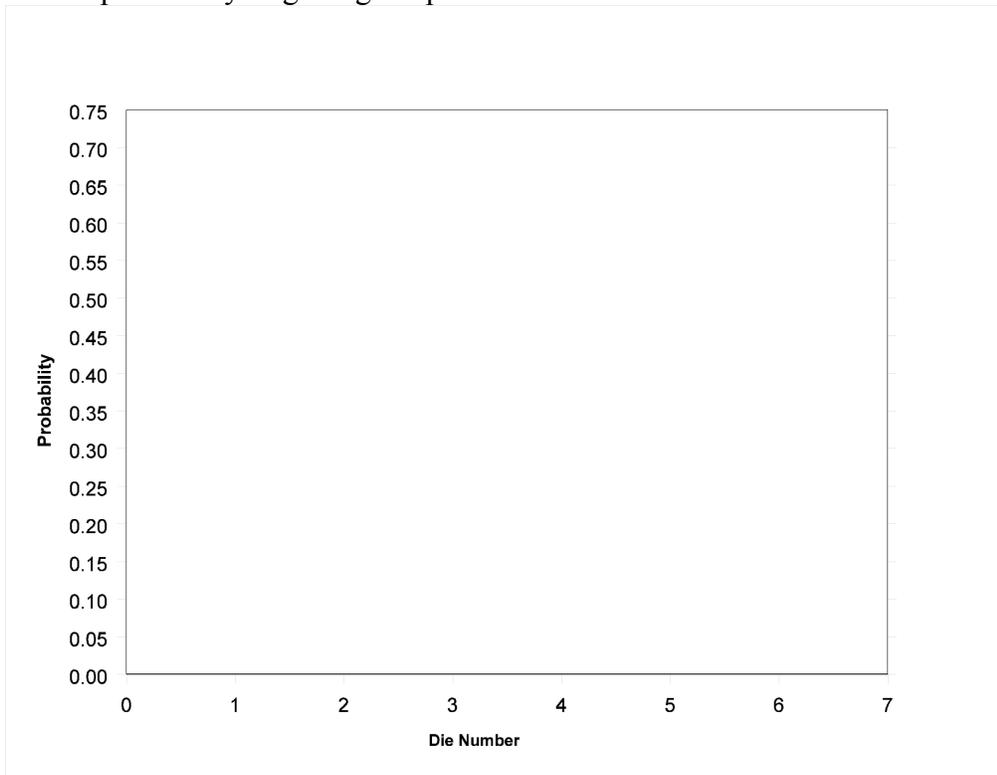
<b>Roll #</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
$\bar{x}$										
<b>Roll #</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>	<b>20</b>
$\bar{x}$										
<b>Roll #</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>	<b>30</b>
$\bar{x}$										
<b>Roll #</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>	<b>40</b>
$\bar{x}$										

- c. Now sort your 40 values of  $\bar{x}$  and complete Table 3 given below.

Table 3

Interval	Number of Rolls	Proportion out of 40 (Probability)
1 but less than 2		
2 but less than 3		
3 but less than 4		
4 but less than 5		
5 to 6		

2. We will now construct two histograms on the top and bottom of this page: Table 1 (single roll of a die) on the top and Table 3 (mean of 4 rolls of a die) on the bottom. Remember, the horizontal axis represents the values of  $x$  (or  $\bar{x}$ ) while the vertical axis represents the probability of getting the particular value.



### Exercise Questions

Recall the premise of the Central Limit Theorem: The mean of a random sample will approximately follow a normal distribution with mean  $\nu$  and standard error  $\frac{\sigma}{\sqrt{n}}$ , regardless of the distribution of the population. The theory requires a sample size of at least 30 if the population distribution is unknown. However, because we know the distribution of the die and this distribution is symmetric, we can get away with a much smaller sample size ( $n=4$ ) and still see how the Central Limit Theorem works.

We will now compare the results of rolling one die versus the experiment you performed: the mean of 4 rolls of a die.

1. Comment on the difference in shape between the top and bottom histograms.
2. Do you believe that there is a Central Limit Theorem effect working with regards to the shapes of the bottom distribution?
3. If the population has a standard deviation of  $\sigma$  and the mean of a random sample has a standard error of  $\frac{\sigma}{\sqrt{n}}$  (according to the Central Limit Theorem), which one of these two values will be lower if  $n > 1$ ? Hence, which one of these two distributions do you expect to have lower variability: the population or the mean of a random sample taken from this population?
4. Looking at the top and bottom histogram, which one appears to have less variance: the population or the mean of the random sample?
5. Do you believe that the Central Limit Theorem is working here with regards to the standard error of the mean?