Using Random Numbers to Introduce the Central Limit Theorem

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Abstract: Random numbers can be used as a quick way to introduce students to the effect of sample size on the shape of the sampling distribution of the sample mean. The parent distribution and effect of the Central Limit Theorem can be visualized as the sample size is changed.

Outline:

A) Statement of Central Limit Theorem
B) Quick Overview of Theoretical Parent and Associated Sampling Distributions for the Sample Mean
C) Illustration of Obtaining Observations
   1) From the Uniform (Parent) distribution
   2) From associated sampling distributions for n=2 and n=4.
D) Comparisons of the Three Distributions
E) Comparisons of Statistics by Theory and for the Data
F) Final Remarks

Practical Considerations:

A) Need access to a random number table.
B) Easier for moderate class sizes. Too small or too large would not be so workable.
C) Though it takes some time, the overall demonstration takes 10 to 15 minutes. Obviously, more if more extensive discussion happens.
D) Good active participation exercise that seems to help students understand that something does happen in the sampling distribution as the sample size changes. They also better understand that each observation in the sampling distribution represents n observations from the parent distribution.

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PowerPoint Screens

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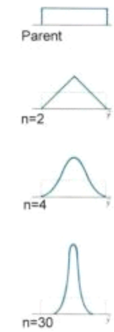
The Central Limit Theorem

Let $\bar{X}$ be the mean of a random sample of size n from a parent population that has mean μ and standard deviation σ. When the sample size, n, is sufficiently large, the sampling distribution of $\bar{X}$ will be approximately normally distributed with mean μ and standard deviation (or standard error) $\sigma/\sqrt{n}$.

Theoretical Distributions



Parent Distribution

23212	74483	36590	0	••
17639	96252	95649	1	••
74197	81962	48443	2	••••
04429	31308	02241	3	••••
59109	88976	46845	4	•••
93458	42161	26099	5	•••
55212	33360	68751	6	•••
25906	64708	20307	7	••
66449	32353	83668	8	•••
96372	50277	15571	9	••••

Sample Mean, n=2

23212	74483	36590	0	
17639	96252	95649	1	•••
74197	81962	48443	2	••••
04429	31308	02241	3	••••
59109	88976	46845	4	••••
93458	42161	26099	5	••••••
55212	33360	68751	6	•••
25906	64708	20307	7	•••••
66449	32353	83668	8	•
96372	50277	15571	9	

Sample Mean, n=4

23212	74483	36590	0	
17639	96252	95649	1	••
74197	81962	48443	2	•••
04429	31308	02241	3	••••••
59109	88976	46845	4	•••••
93458	42161	26099	5	••••••••••
55212	33360	68751	6	••••
25906	64708	20307	7	
66449	32353	83668	8	•
96372	50277	15571	9	

	n=1	n=2	n=4
0	••		
1	••	•••	••
2	••••	••••	•••
3	••••	••••	••••••
4	•••	••••	•••••
5	•••	••••••	••••••••••
6	•••	•••	••••
7	••	•••••	
8	•••	•	•
9	••••		

n=1

0	••
1	••
2	••••
3	••••
4	•••
5	•••
6	•••
7	••
8	•••
9	••••

Pop	Sample
$\mu = 4.5$	$\bar{x} = 4.70$
$\sigma^2 = \frac{99}{12}$	$s^2 = 8.14$

$$DU(0,N): \mu = \frac{N}{2}; \sigma^2 = \frac{N(N+2)}{12}$$

n=2

0	
1	•••
2	••••
3	••••
4	••••
5	••••••
6	•••
7	•••••
8	•
9	

Pop	*Sample*
$\mu = 4.5$	$\bar{x} = 4.51$
$\sigma^2 = \frac{99}{24}$	$s^2 = 4.2($

n=4

0	
1	••
2	•••
3	••••••
4	•••••
5	••••••••••
6	••••
7	
8	•
9	

Pop	*Sample*
$\mu = 4.5$	$\bar{x} = 4.44$
$\sigma^2 = \frac{99}{48}$	$s^2 = 2.56$

Final Remarks

- Theory *predicts* "reality"
- Theory *becomes* reality
- Concept becomes less of a mystery
- Students are involved
- Activity is fairly quick and very easy to do