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MAD about inference:

A simple (but rigorous!) alternative to the t-test for data investigations

eCOTS 2026: Poster #51

Thursday, June 18, 1:15-2:00 pm, ET

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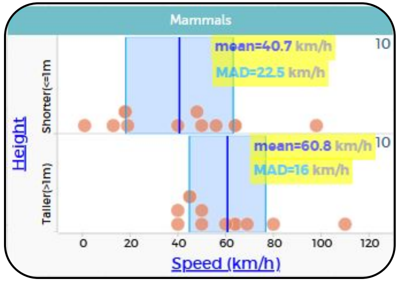
Background and Motivation

- MAD is a developmental precursor to the SD (e.g. CCSSM-6.SP.B.5):
    - Conceptually simpler; more intuitive; prior exposure to MAD
    - A bridge between informal and formal inferential reasoning

$$\text{MAD} = \frac{\sum |\text{observed} - \text{mean}|}{n}, \qquad \text{SD} = \sqrt{\frac{\sum (\text{observed} - \text{mean})^2}{n-1}}$$

- MAD is used to "express the difference between two groups as a multiple of a measure of variability" (CCSSM-7.SP.B.3)

- And *can* be used to "gauge the variation in estimates or predictions" (i.e., confidence intervals) (CCSSM-7.SP.A.2)



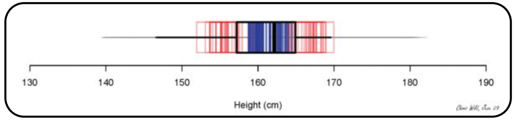


Image credit: https://www.stat.auckland.ac.nz/~wild/talks/09.USCOTS.html

Background and Motivation

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$$\text{MAD} = \frac{\sum |\text{observed} - \text{mean}|}{n}, \quad \text{SD} = \sqrt{\frac{\sum (\text{observed} - \text{mean})^2}{n-1}}$$

$$k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{d_p}$$

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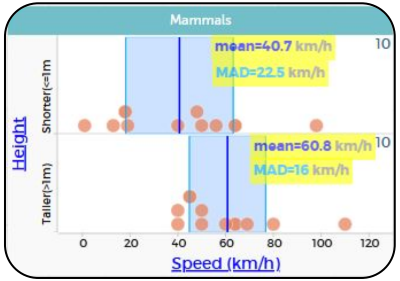

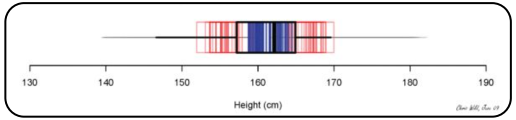

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Equivalence between k and t

The MAD can be used for formal inference because the **k-statistic** is equivalent to a **scaled t-statistic**.

$$k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{d_p} \approx t \cdot \sqrt{\frac{\pi}{n-1}}$$

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Equivalence between k_{two} and t

Start with the **two-sample** *t*-statistic:

$$\boxed{t =} \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{(\bar{x}_1 - \bar{x}_2)}{\boxed{s_p}\sqrt{\frac{2}{n}}}$$

Sub $\boxed{s \approx d\sqrt{\frac{\pi}{2}\left(\frac{n}{n-1}\right)}}$ (Revets, 2009):

$$\approx \frac{(\bar{x}_1 - \bar{x}_2)}{\boxed{\left(d_p\sqrt{\frac{\pi}{2}\left(\frac{n}{n-1}\right)}\right)}\sqrt{\frac{2}{n}}}$$

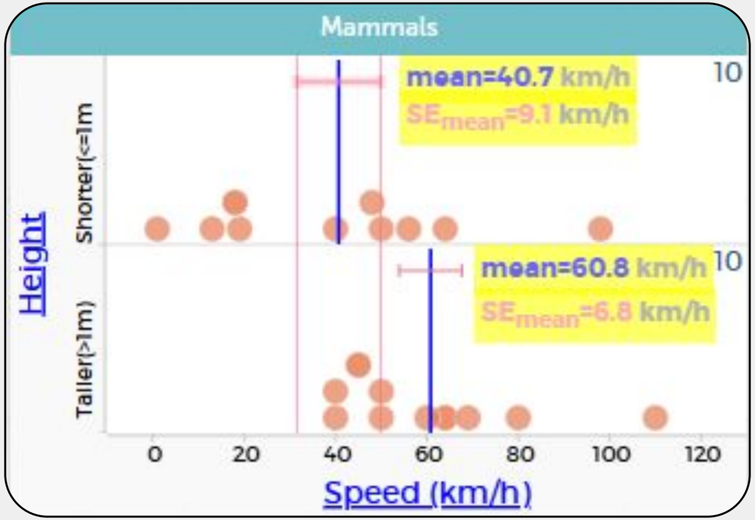
Substitute $k = \frac{\bar{x}_1 - \bar{x}_2}{d_p}$ and simplify:

$$\boxed{= \frac{k}{\sqrt{\frac{\pi}{n-1}}}}$$

Results: 1 $\boxed{t \approx k \cdot \sqrt{\frac{n-1}{\pi}}}$ ⇔ 2 $\boxed{t \cdot \sqrt{\frac{\pi}{n-1}} \approx k}$

Example

- Do taller (>1m) land-mammals tend to run faster than shorter ones? (source data)



Example

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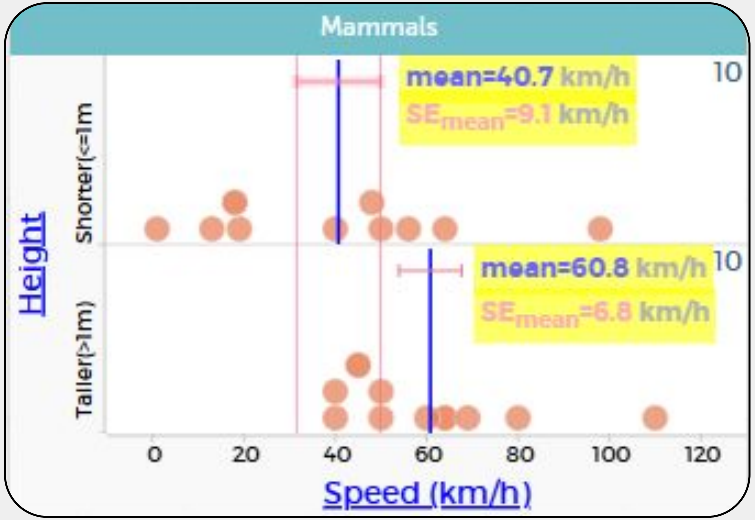
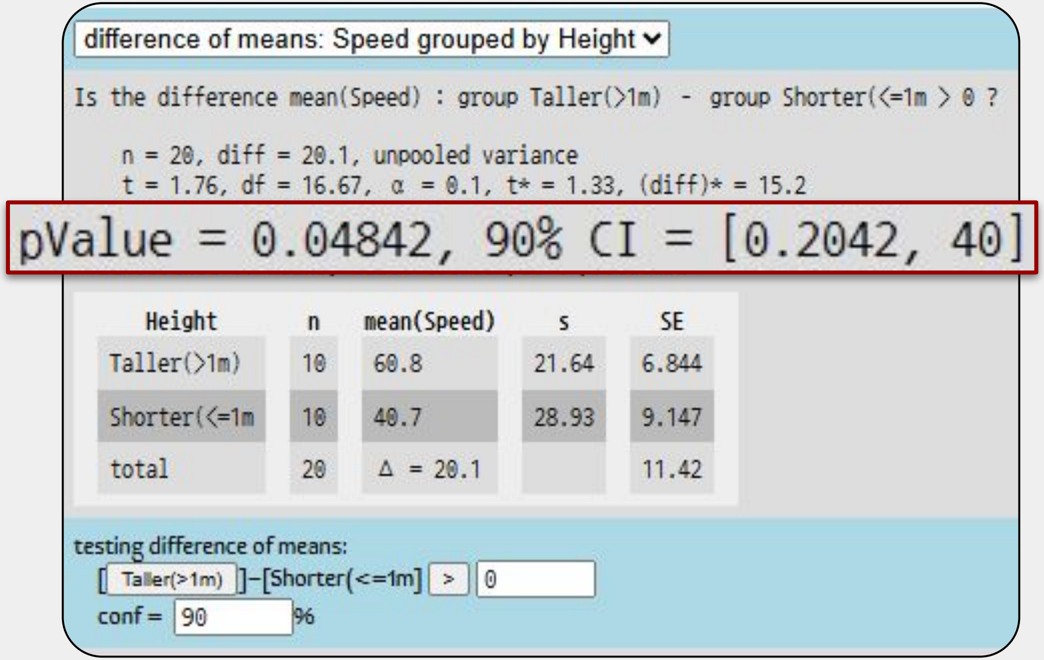


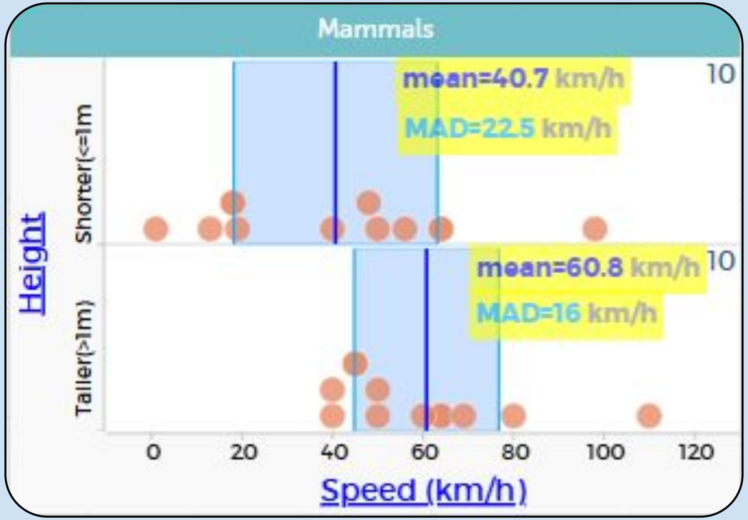
- t-test (r.t.) says:
  p = 0.048, 90% CI = [0.2, 40]



t-test (r.t): $p = 0.048$

Example

- Do taller (>1m) land-mammals tend to run faster than shorter ones? ([source data]())



MAD-based *p*-value:

$$k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{MAD} \rightarrow \boxed{k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{d_p}}$$

$$d_p = \frac{1}{2}(22.5 + 16) = 19.25$$

$$k_{two} = \frac{60.8 - 40.7}{d_p} = 1.044$$

1

$$t = 1.044 \cdot \sqrt{\frac{9}{\pi}} = 1.767$$

$$P(t_{18} > 1.767) = 0.0471$$

k_{two} p-value calculator↗

Exam

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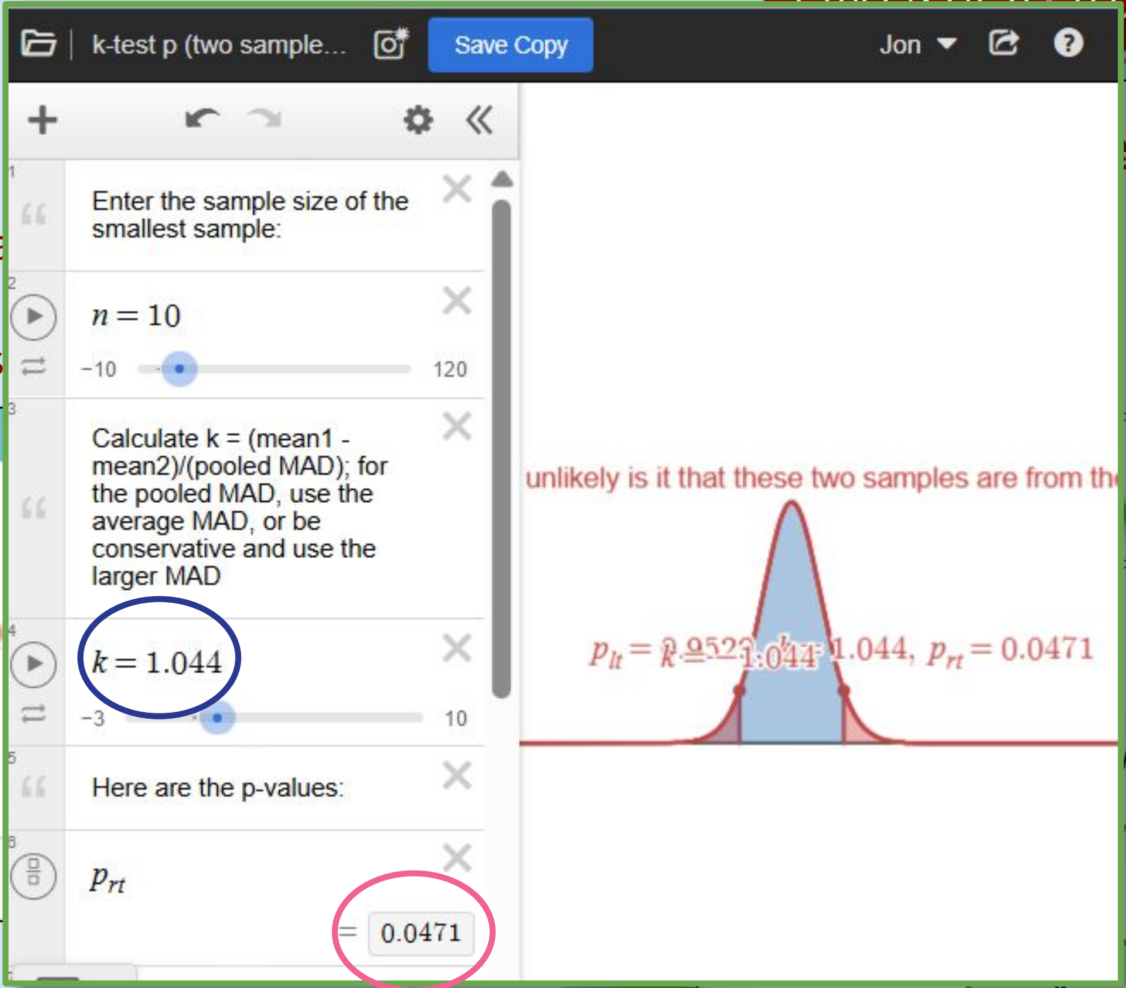


t-test (r,t): p = 0.048

$$k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{d_p}$$

19.25

1.044

$\sqrt{\frac{9}{\pi}} = 1.767$

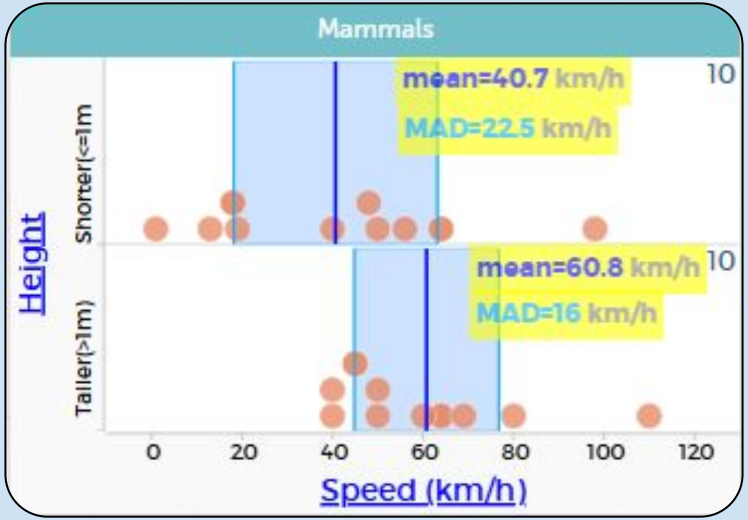
$767) = 0.0471$

[k_{two} p-value calculator]

t-test (r.t): 90% CI = [0.2, 40.0]

Example

- Do taller (>1m) land-mammals tend to run faster than shorter ones? (source data)



MAD-based CI:

$$t_{18,\,0.10} = 1.734$$

$$d_p = \frac{1}{2}(22.5 + 16) = 19.25$$

2

$$\Rightarrow \; k^* = 1.734 \cdot \sqrt{\frac{\pi}{9}} = 1.024$$

$(k^*)(d_p)$ is the margin of error:

$$\Delta\bar{x} \pm (k^*)(d_p) = 20.1 \pm (1.024)(19.25)$$

$$= [0.4, 39.8]$$

k_{two}-crit calculator

t-test (r.t): 90% CI = [0.2, 40.0]

Example

- Do
  te
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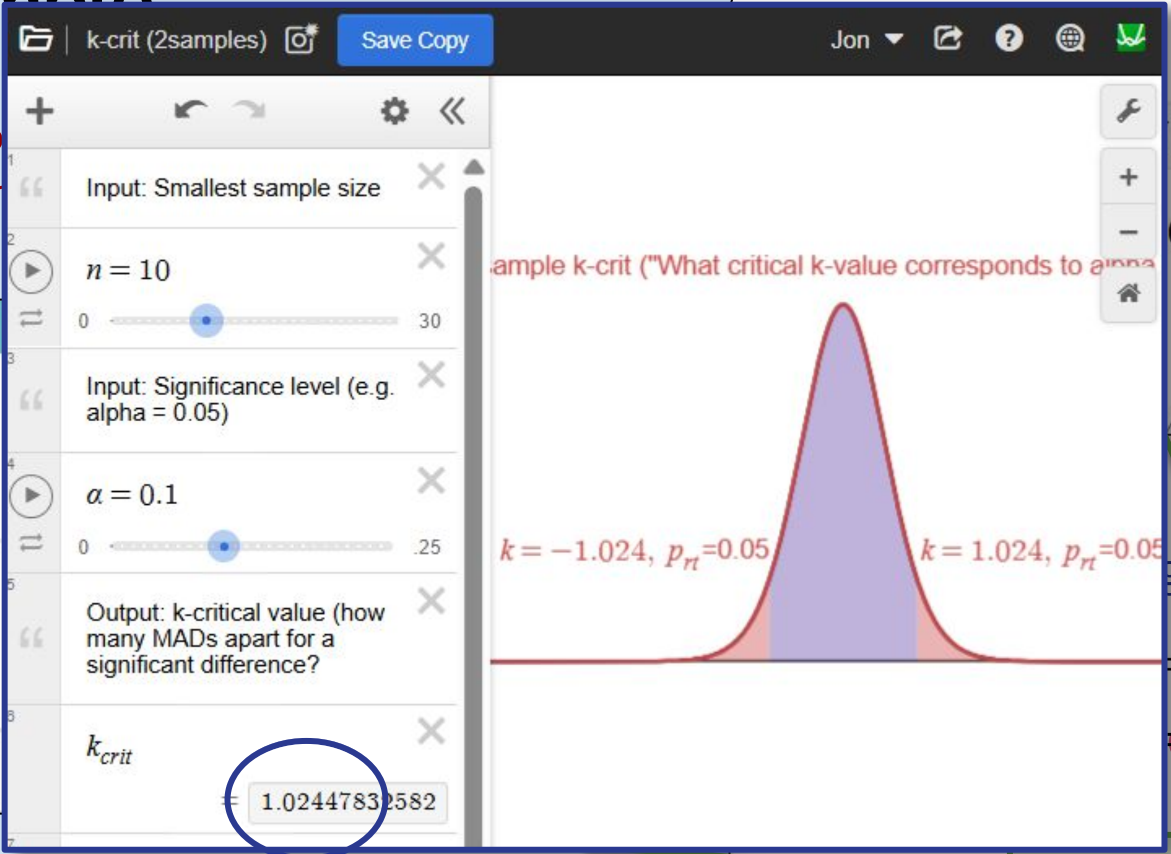


34

$6) = 19.25$

$4 \cdot \sqrt{\frac{\pi}{9}} = 1.024$

gin of error:

$= 20.1 \pm (1.024)(19.25)$

$= [0.4, 39.8]$

[k_{two}-crit calculator](#)

Equivalence between k_{one} and t

The MAD can be used for formal inference because the k-statistic is equivalent to a scaled t-statistic.

Two-sample case:

$$k_{two} = \frac{\bar{x}_1 - \bar{x}_2}{d_p} \approx t \cdot \sqrt{\frac{\pi}{n-1}}$$

1 $$\boxed{t \approx k \cdot \sqrt{\frac{n-1}{\pi}}}$$

2 $$\boxed{t \cdot \sqrt{\frac{\pi}{n-1}} \approx k}$$

One sample case:

$$t = \frac{\bar{x} - \mu}{SE} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$k_{one} = \frac{\bar{x} - \mu}{d} \approx t \cdot \sqrt{\frac{\pi}{2n-2}}$$

3 $$\boxed{t \approx k \cdot \sqrt{\frac{2n-2}{\pi}}}$$

4 $$\boxed{t \cdot \sqrt{\frac{\pi}{2n-2}} \approx k}$$

Equivalence between k_{one} and t

Start with the **one-sample** *t*-statistic:

$$\boxed{t} = \frac{\bar{x}-\mu}{SE} = \frac{\bar{x}-\mu}{\boxed{s}/\sqrt{n}}$$

Sub $\boxed{s \approx d\sqrt{\frac{\pi}{2}\left(\frac{n}{n-1}\right)}}$ (Revets, 2009):

$$= \frac{\bar{x}-\mu}{d\boxed{\sqrt{\frac{\pi \cdot n}{2n-2}} \cdot \frac{1}{\sqrt{n}}}}$$

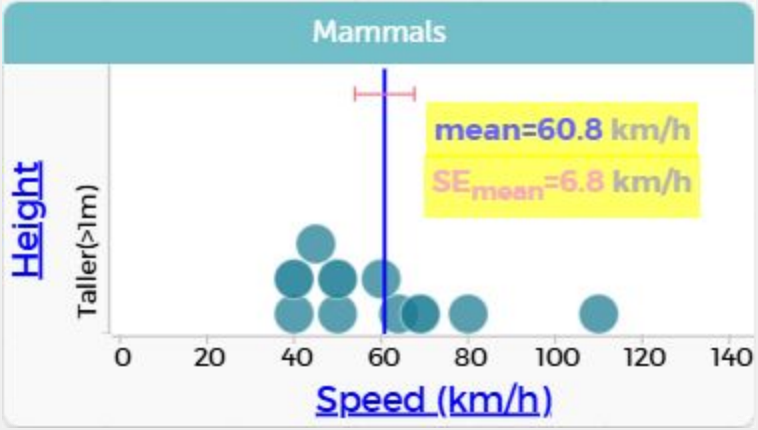
Substitute $k = \frac{\bar{x}-\mu}{d}$ and simplify:

$$= \boxed{\frac{k}{\sqrt{\frac{\pi}{2n-2}}}}$$

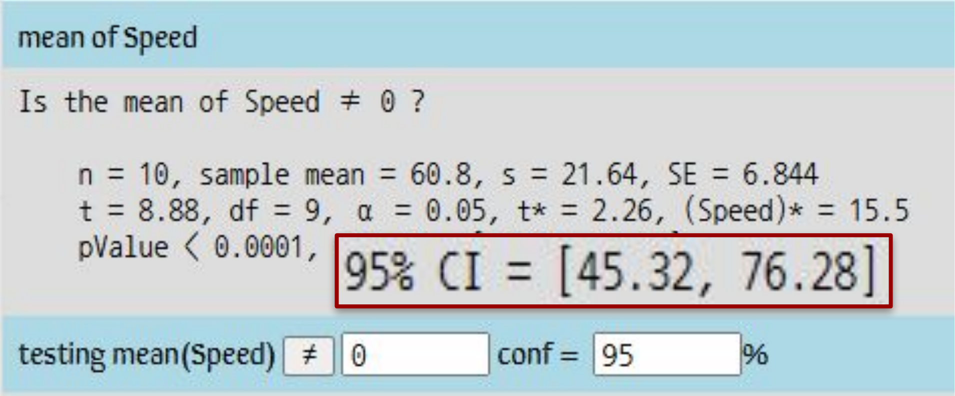
Results: 3 $\boxed{t \approx k \cdot \sqrt{\frac{2n-2}{\pi}}}$ ⇔ 4 $\boxed{t \cdot \sqrt{\frac{\pi}{2n-2}} \approx k}$

Example

- Estimate the mean speed of taller land-based mammals. (source data)



- t-test says:
  95% CI = [45.3, 76.3]


mean of Speed

Is the mean of Speed ≠ 0 ?

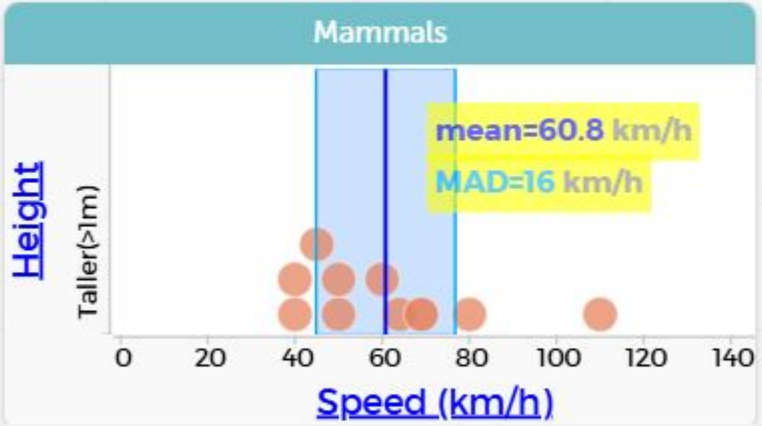
n = 10, sample mean = 60.8, s = 21.64, SE = 6.844
t = 8.88, df = 9, α = 0.05, t* = 2.26, (Speed)* = 15.5
pValue < 0.0001, 95% CI = [45.32, 76.28]

testing mean(Speed) ≠ 0 conf = 95 %

t-based CI: [45.3, 76.3]

Example

- Estimate the mean speed of taller land-based mammals. ([source data](#))



MAD-based CI:

$$t_{9,0.05} = 2.262$$

4

$$\Rightarrow k^* = 2.262 \cdot \sqrt{\frac{\pi}{18}} = 0.945$$

(k*)(d) is the margin of error:

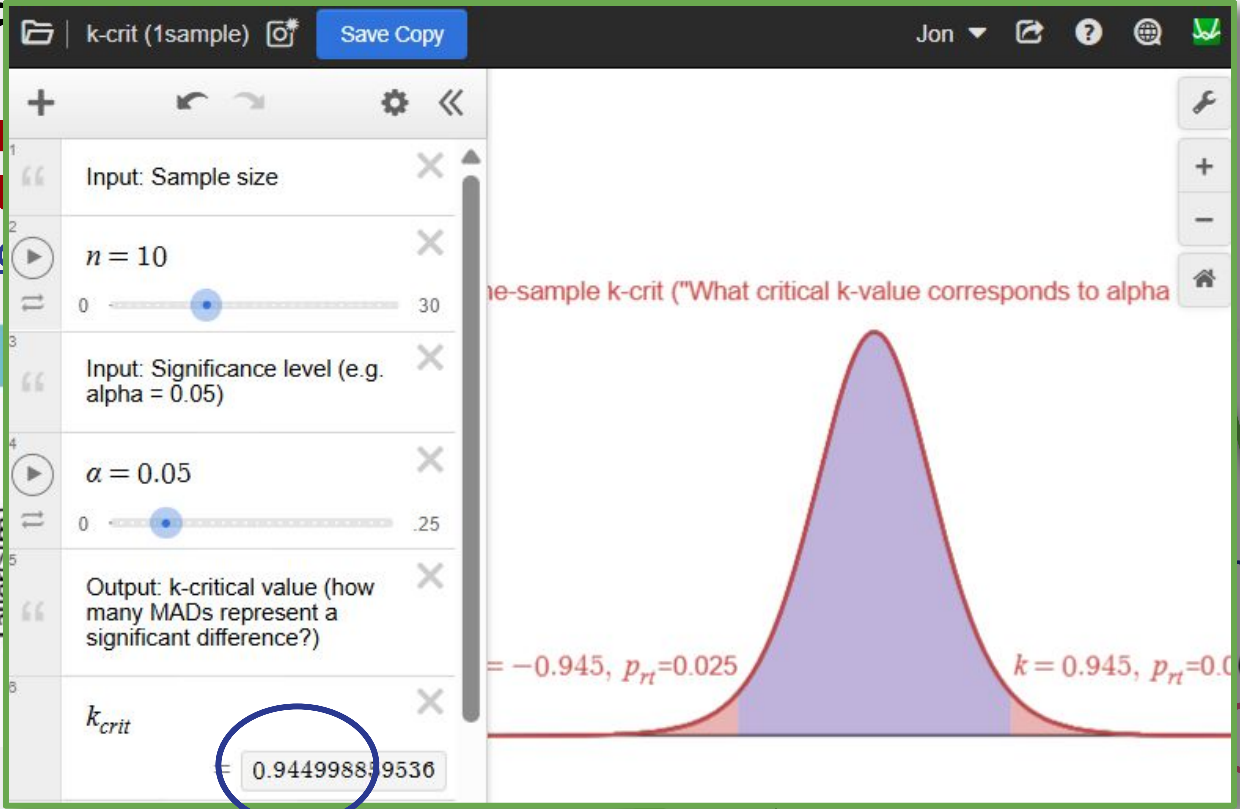
$$\bar{x} \pm (k^*)(d) = 60.8 \pm (0.945)(16)$$

= [45.7, 75.9] km/h

[k_{one}-crit calculator](#)

t-based CI: [45.3, 76.3]

Example



Height

$\sqrt{\frac{\pi}{18}} = 0.945$

of error:

$60.8 \pm (0.945)(16)$

[45.7, 75.9] km/h

k_{one}-crit calculator

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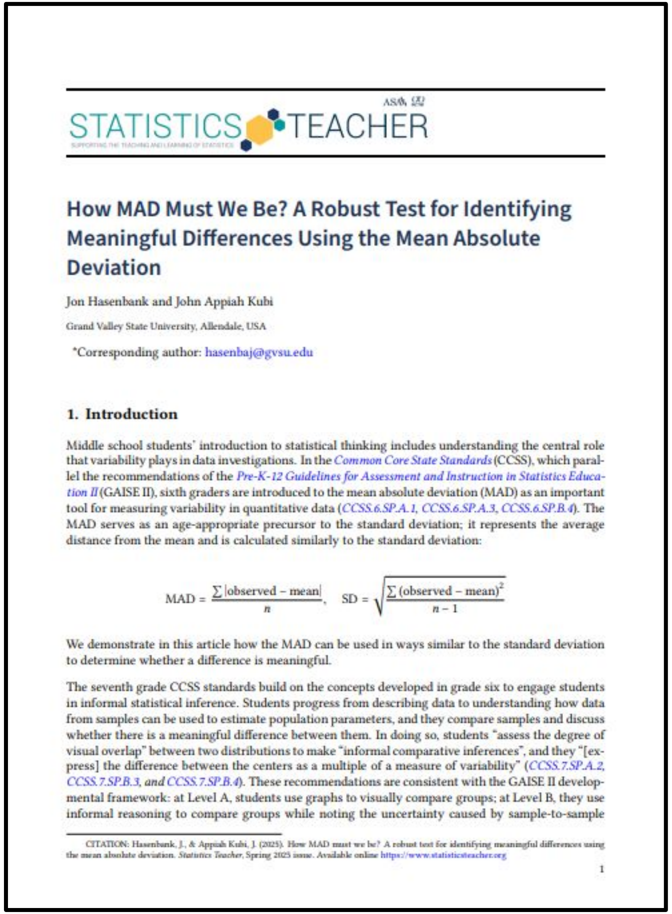
k-Calculators and Conclusions

Desmos CI-calculator tools:
https://faculty.gvsu.edu/hasenbaj/

1- or 2-sample	Get p-value	Get critical value
1-sample	Snapshot | Source (2026-01-07)	Snapshot | Source (2026-04-10)
2-sample	Snapshot | Source (2026-02-06)	Snapshot | Source (2026-01-07)

To learn more, stop by our poster on Thursday 6/18 or download our 2025 article at StatisticsTeacher.org



STATISTICS TEACHER

SUPPORTING THE TEACHING AND LEARNING OF STATISTICS

How MAD Must We Be? A Robust Test for Identifying Meaningful Differences Using the Mean Absolute Deviation

Jon Hasenbank and John Appiah Kubi

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1. Introduction

Middle school students' introduction to statistical thinking includes understanding the central role that variability plays in data investigations. In the *Common Core State Standards* (CCSS), which parallel the recommendations of the *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II* (GAISE II), sixth graders are introduced to the mean absolute deviation (MAD) as an important tool for measuring variability in quantitative data (*CCSS.6.SP.A.1, CCSS.6.SP.A.3, CCSS.6.SP.B.4*). The MAD serves as an age-appropriate precursor to the standard deviation; it represents the average distance from the mean and is calculated similarly to the standard deviation:

$$\text{MAD} = \frac{\sum|\text{observed} - \text{mean}|}{n}, \quad \text{SD} = \sqrt{\frac{\sum(\text{observed} - \text{mean})^2}{n-1}}$$

We demonstrate in this article how the MAD can be used in ways similar to the standard deviation to determine whether a difference is meaningful.

The seventh grade CCSS standards build on the concepts developed in grade six to engage students in informal statistical inference. Students progress from describing data to understanding how data from samples can be used to estimate population parameters, and they compare samples and discuss whether there is a meaningful difference between them. In doing so, students "assess the degree of visual overlap" between two distributions to make "informal comparative inferences", and they "[express] the difference between the centers as a multiple of a measure of variability" (*CCSS.7.SP.A.2, CCSS.7.SP.B.3, and CCSS.7.SP.B.4*). These recommendations are consistent with the GAISE II developmental framework: at Level A, students use graphs to visually compare groups; at Level B, they use informal reasoning to compare groups while noting the uncertainty caused by sample-to-sample

CITATION: Hasenbank, J., & Appiah Kubi, J. (2025). How MAD must we be? A robust test for identifying meaningful differences using the mean absolute deviation. *Statistics Teacher*, Spring 2025 issue. Available online https://www.statisticsteacher.org

References and Additional Reading (1 of 2)

American Statistical Association. (2020). *Pre-K-12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II): A Framework for Statistics and Data Science Education.* Retrieved from: https://www.amstat.org/asa/files/pdfs/GAISE/GAISEIIPreK-12_Full.pdf

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BONUS CONTENT

Simulations showing the very strong correlation between the sample MAD and SD (for a given n)

These slides: gvsu.edu/s/3EU

Estimating k^* from t^* via simulations

Initial idea: Estimate *k*-critical values via **simulation.**

1. Plot *t* vs. *k* for many random samples (fixed *n* & α).

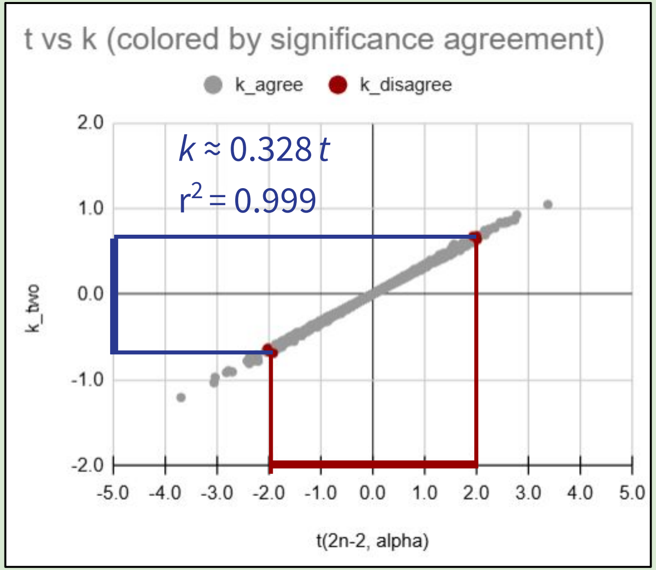
   Insight → $k \approx 0.328\, t$ ($r^2 = 0.999$) ←← for *n* = 30!

2. Use t^* to estimate k^* (critical values).

   (Repeat for other n and α)

E.g. for $n = 30$ and $\alpha = 0.05$:

$|t^*| \leq 2.002 \Leftrightarrow |k^*| \leq 0.657$



Simulations yield the following thresholds for $\alpha = 0.05$:

- for ***n* = 15**, **k* = ~1** (so a 1-MAD difference is meaningful)
- for ***n* = 30**, **k* = ~⅔**
- for ***n* = 50**, **k* = ~½**