Culturally Responsive Teaching of Probability: La Lotería & Toma Todo

activity presented 5/25/22 at the 6th Electronic Conference on Teaching Statistics (Track #2: Diversity, Inclusion, and Social Justice)

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This activity was developed in the context of connecting with the underserved and rapidly-growing Latinx/Mexican-American/Hispanic population. It gives such students authentic and engaging context for applying principles of probability and serves to honor and acknowledge their culture (and educate students less familiar with that culture). The two parts involve two beloved popular games from Mexican-American culture: La Lotería and Toma Todo.

For each part, we provide learning objectives and questions that can be used for an informal exploration or a formal assessment, and offer solutions. This lesson has been class-tested in in-person classes at a Hispanic Serving Institution and there has also been a degree of peer vetting in that pieces of this lesson have been adapted from bits of pieces the author has published (see REFERENCES at the end). This activity can be used in an introductory statistics (or statistics literacy) course at either the university, two-year college, or high school level. Whether 1 or 2 class periods are needed depends on (1) the length of the period, (2) whether this activity is being used to introduce, apply, or review concepts, (3) students' background, and (4) whether the class takes time to first play the game vs. just learns enough about the game to answer the questions.

Part One: La Lotería

Background: Many sources say la lotería (Spanish for "the lottery") originated in Italy in the 15th century, and came (via Spain) to Mexico in 1769. Lotería is like bingo, but each player's board (also called a *tabla*) is 4x4 (not 5x5) with no "free" space and has colorful, vivid images instead of numbers. Each 16-image board (e.g., see Figure 1 below from Ramirez & McCollough, 2012) is different and there are 54 different images in the set of cards that are called out one-at-a-time.

The caller (*cantor*, Spanish for "singer") running the game selects cards one-at-a-time at random and may not just call out the name of an image (for an example from Figure 1: "*La Escalera*" [the ladder]), but instead sing out a riddle or give a poetic description (e.g. "Súbeme paso a pasito. No quieras pegar brinquitos. [Climb me step by step. You don't want to hop up.]"). The first player to get 4 tokens in one of the patterns of Figure 1 and call out "¡Lotería!" wins. (Note: some versions of the game may choose other ways to win such as "blackout.")

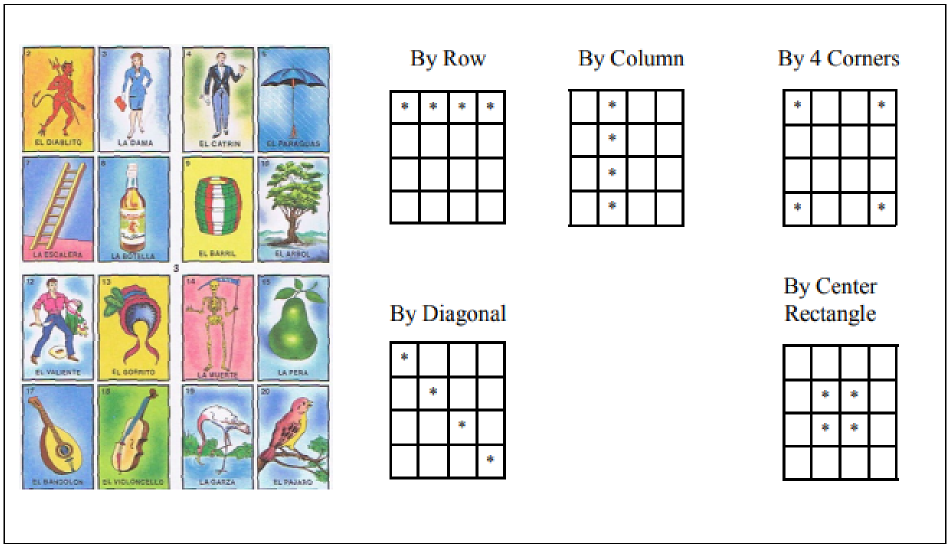


Figure 1. Sample "La Lotería" board and types of winning moves.

Materials: First, have on hand plenty of tokens (say, 10-15 times the number of players) for players to place on the board. The tokens are traditionally raw pinto beans, but could also be uncooked corn, nuts, raisins, wrapped hard candies, pebbles, plastic chips, buttons, etc. (The number of tokens will also cover the Toma Todo activity, of course.) The game boards (*tablas*) and deck of cards are available for under $10 in physical and online gift shops that sell Mexican cultural items, and there are also plenty of websites on how to make your own tablas and cards. If it is not practical to take time to play it in class, students could be invited to play outside of class (e.g., using an online version such as https://www.google.com/doodles/celebrating-loteria) and watch a short video about it (e.g., https://www.youtube.com/watch?v=QbAuUVe0W9c or https://www.youtube.com/watch?v=QMxUIhEb84A). Then students can be asked to answer a series of questions while having in view Figure 1 from Ramirez & McCollough (2012).

Learning Objectives: Question 1 uses learning objectives for the multiplication rule for independent events as well as the complement rule. Questions 2 and 4 involve the learning objective of identifying possible outcomes of a probability situation. Question 3 has the learning objective of finding a probability by first calculating a combination.

1. What's the probability that the first card called is not on your 4x4 tabla?
   What's the probability that neither of the first two cards called is on your tabla?

Solution: 16 of 54 cards ARE on your tabla, so the complement rule says
P(1^{st} card called is not on your tabla) = 1 – P(1^{st} card is on your tabla) = 1 – (16/54) = 38/54.
Since cards are drawn independently without replacement, the probability that both the first and second cards are not on your board is (38/54)*(37/53).

2. Based on Figure 1, how many ways are there for a player to win?

Solution: Figure 1 helps us see that there are 12: 4 ways to win by "row", 4 ways by "column", 2 ways by "diagonal", 1 way by "4 corners" and 1 way by "center rectangle"'

3. What's the probability that you have a win after the caller calls exactly 4 cards?

Solution: The probability of this (very unlikely) event can be obtained in more than one way. We can take the answer from Question #2 and divide it by the ways the caller can choose 4 cards from 54. In other words, 12/ C(54,4) = 12/316251 = .000038, which is less than 1 in 26,000. Another way to look at it is to find the probability that the first four cards called by the caller happen to all be on the player's game board, and then multiply that answer by the probability that those 4 cards happen to be in a geometric pattern that is (out of all ways 4 spaces can be chosen on a 4x4 board) one of the 12 ways of winning. And so, we obtain [(16/54)(15/53)(14/52)(13/51)] * (12/ C(16,4)), which yields the same (tiny) answer!

4. What's the smallest number of cards (out of 54) that the caller could call before your 4x4 board MUST win?

Solution: Suppose 12 cards have been called that are on your game board, as shown by X's in the following figure (from Lesser, 2013). As you see, it is possible to cover these 12 spaces and yet not have one of the winning combinations. (A way to derive such a figure is to first put X's in 3 of the 4 corners. The empty corner will mean you have to leave an empty cell in one of the innermost four cells that is not on its diagonal. That latter empty cell in turn means you can fill in the rest of the row and column it is in. It will then be clear which two of the remaining cells must be left empty.) Now, assume the 4 uncovered spaces on your game board are the only 4 cards that have not yet been called in the deck of 54 cards. Then, this means that 50 cards have been called so far, and the very next card must produce a win, and so the answer to the question is 51.

X	X	X	
	X	X	X
X	X		X
X		X	X

Enrichment/Extensions:

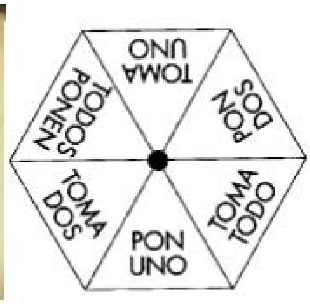
- How is the layout of open spaces in the above diagram similar to and different from a Latin square design?
- What is the expected number of cards called before you would be able to shout "¡Lotería!" (independent of other players)?
- from https://fivethirtyeight.com/features/can-you-win-the-loteria/: "you and your friend Christina decide to face off in a friendly game of Lotería. What is the probability that either of you ends the game with an empty grid, i.e. none of your images was called? How does this probability change if there were more or fewer unique images? Larger or smaller player grids?

Part Two: Toma Todo

Background: Toma Todo (sometimes called *la pirinola*, which is the name of the spinning top (or *topa*) used in the game) is a game common in México, and some other parts of Latin America. We share a composite of the most standard versions of the children's game (i.e., we are not considering variations some adults may have later modified for gambling or drinking).

A round consists of each player getting one turn to spin the pirinola. To launch the game, the players determine the order they will spin within a round and sit in a circle in that order. Then decide how many tokens each player starts with (e.g., 10) and the initial size of the pile in the center called the "pot." Decide what it will mean to "win"—while you could play until one person has all of the tokens, it will be more practical and fast to just see who is ahead after a fixed number of rounds. Now start the first round. If the pot gets down to 0 or 1 token, every player puts 2 tokens into the pot. A player running out of tokens is eliminated from the game.

Materials: The pirinola is a six-sided top available in physical and online gift shops that sell Mexican cultural items. A top made of wood (see photo below from Lesser, 2010) can be up to $10 while a plastic one is only $2. There are also virtual apps for this game, and the top can also be handmade by sticking a toothpick or pencil halfway through a hexagonal piece of cardboard divided into 6 triangles (as shown below on the right; templates can be found online). An alternative to a top is using a 6-region spinner or a cubical die.



The following table (in descending order of how good the outcome is for the person who just spun) lists what Spanish words are printed on each of the six sides of the pirinola, along with what it means in terms of the game. Basically, you need to learn just 3 words:

toma = take (from the pot), pon(en) = put (into the pot), todo(s) = all

Face-up side	Toma todo	Toma 2	Toma 1	Todos ponen	Pon 1	Pon 2
Action by player who just spun	Take all chips from pot	Take 2 chips from pot	take 1 chip from pot	ALL players put 2 chips in pot	Put 1 chip in pot	Put 2 chips in pot

Assume N players are playing. Let's say that each player starts off with, say, a dozen chips and that the pot initially starts off with P chips in it. [Note: to make the problem easier or more accessible to students with less mathematical background, you could change P to a specific number – say, 10 – and choose N to be, say, 5. Another option is to just let the initial pot P = 2N, since that is the "reset" pot when it gets emptied by someone spinning "toma todo".]

Learning Objectives: The learning objectives related to questions 1-4 mainly involve calculating expected value and articulating a sample space. Question 5 deals with the learning objective of recognizing when events are independent.

For these questions, suppose the first player to spin is Alonzo, and the second player is Elena.

1.) What is the probability that Alonzo will have an increase in the number of chips he has after he takes the first turn? Show your work and reasoning.

Solution: of the 6 possible outcomes (which are equally-likely), 3 of them involve a gain, so the answer is $3/6 = \frac{1}{2}$.

2.) What is the expected value of the number of chips that Alonzo will win on the first turn? Show your work and reasoning. Is this a value that could actually occur?

Solution: $(1/6)(1) + (1/6)(2) + (1/6)(P) + (1/6)(-1) + (1/6)(-2) + (1/6)(-2) = (P-2)/6$, which is always nonnegative since the rules do not allow the pot before a spin to be less than 2. For $P = 10$, this would be an expected gain of 4/3 chips. Note that the result of a spin must be an integral value, so 4/3 is never an actual value that could occur, which is a nice reminder in general about the expected value (analogous to how the mean of a dataset need not be a value of that dataset).

3.) Considering the 6 possible outcomes that can happen with Alonzo's (and the game's) first turn, what is the expected value of the number of chips that will be in the pot right before Elena takes the game's second (and her first) turn? Show your work and reasoning.

Solution: The 6 outcomes are P-1, P-2, 2N, P+1, P+2, P+2N. The expected value (the mean of these outcomes) is $(5P + 4N)/6$, which for $P=10$ and $N=5$ equals $70/6 = 11\ 2/3$ chips, which is larger than the original pot of 10. For more generality, students can show algebraically that $(5P + 4N)/6 \geq P$ as long as $P \leq 4N$.

4.) Is the expected value of the number of chips that Elena wins with her first turn (i.e., the game's second turn) less than, greater than, or equal to the correct answer to Question #2? Show your work and reasoning.

Solution: We can answer the question with intuition because the "toma todo" outcome is the only one that depends on the size of the pot and if we know (from Question #3) that the expected size of the pot before Elena spins is larger than it was when Alonzo spun, we can conclude that the answer is "greater than" and that the second player has an advantage over the first player. Of course, we can also take a more formal approach by writing out the sample space (providing values for P and/or N, if desired) faced by Elena with this table:

Outcome for the player that takes the game’s second turn		Result of game’s second spin					
		Toma 1	Toma 2	Toma todo	Pon 1	Pon 2	Todos ponen
Result of game’s first spin	Toma 1	+1	+2	P-1	-1	-2	-2
	Toma 2	+1	+2	P-2	-1	-2	-2
	Toma todo	+1	+2	P	-1	-2	-2
	Pon 1	+1	+2	P+1	-1	-2	-2
	Pon 2	+1	+2	P+2	-1	-2	-2
	Todos ponen	+1	+2	P+2N	-1	-2	-2

5.) Complete the table below, where event A = “Alonzo spins ‘toma 2’ on his first turn.”

Event B	Are A & B independent?
Alonzo spins ‘toma 2’ on his second spin	
Alonzo spins ‘toma 1’ on his second spin	
Alonzo spins ‘toma 1’ on his first spin	
Elena spins ‘toma 2’ on her first spin	

Solution: Answers are yes, yes, no, yes, respectively.
To justify the “no” on the penultimate row, note P(A|B) = 0, which is not equal to P(A) = 1/6.

Enrichment/Extensions:

- Explore the rules of the game of dreidel (a 4-sided top used in Jewish culture during the holiday of Hanukkah) and see which of the first two players has the advantage.
- How might you change the rules of dreidel (or Toma Todo) so that the order players spin does not give anyone an advantage?
- What other put-and-take spinning top games around the world can you explore (e.g., see http://www.dreidelfun.com/2008/10/dreidel-around-the-world.html)?

NOTE:

continue the dialogue in the CAUSE Slack group **causeweb.org/slack**, using this channel:

#ecots22-2-diversity-inclusion-social-justice-in-data-science-and-statistics

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…..on overall DEI work I've done:

https://larrylesser.com/ell-scholarship/