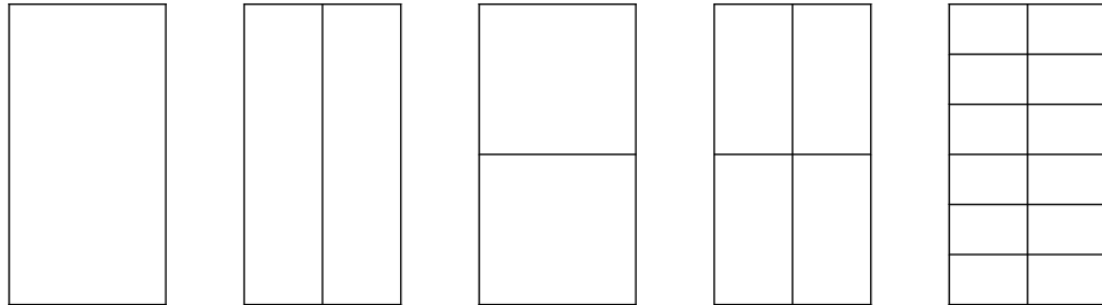


Cobb's Factor Diagrams for Effectively Introducing ANOVA

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The modern* student

- Does not think in terms of algebra
 - Thinks concretely rather than abstractly
 - Thinks visually rather than verbally
- * Not necessarily “modern” developments

Algebraic two-way ANOVA model

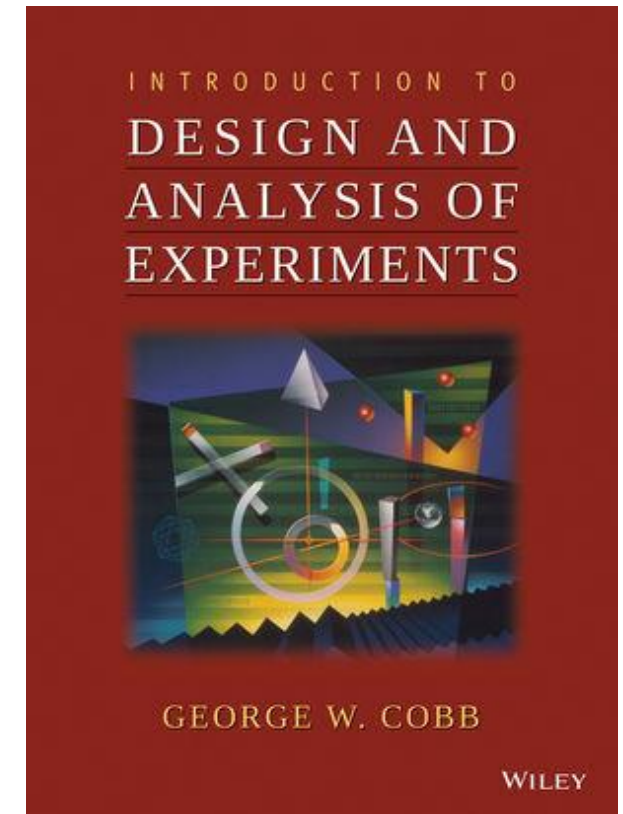
$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

- “Although I believe that many important ideas can be expressed more compactly, and sometimes more precisely, using carefully chosen symbols, I do not accept the defeatist attitude that you can’t learn to design and analyze experiments unless you learn a lot of algebra.”

- George Cobb

Cobb's Design of Experiments book

- Cobb, G. W. (2008) *Introduction to Design and Analysis of Experiments*. Wiley. ISBN: 9780470412169
- Uses factor diagrams to
 - Illustrate data structures
 - Find estimated effects
 - Degrees of freedom
 - ANOVA table
 - Expected Mean Squares



“Pig out” experiment



4 diets

		Antibiotics	
		0 mg	40 mg
Vitamin B12	0 mg	1: Control	3: Antibiotics
	5 mg	2: B12	4: Both

Data table

Response: Weight gain (lbs/week)

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54

Factor diagram idea

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54

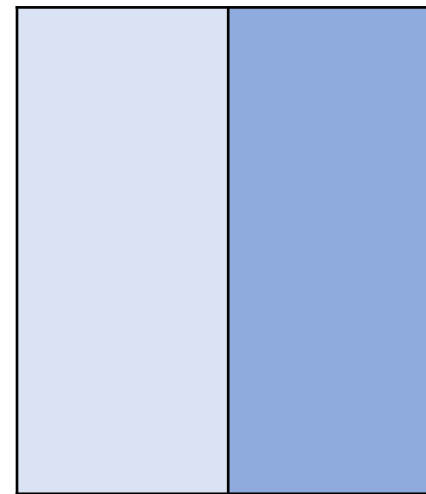


Factor diagram idea

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54



Factor:
Antibiotics

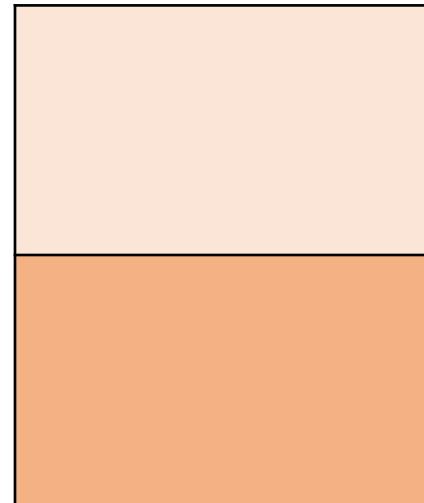


Factor diagram idea

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54



Factor:
B12

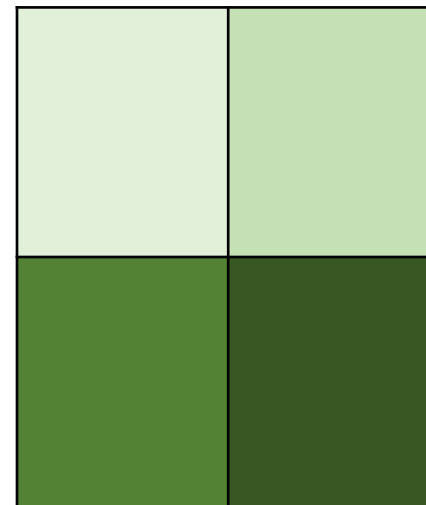


Factor diagram idea

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54



Factor:
Interaction



Factor diagram idea

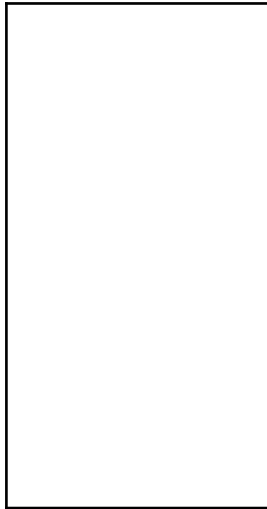
		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54



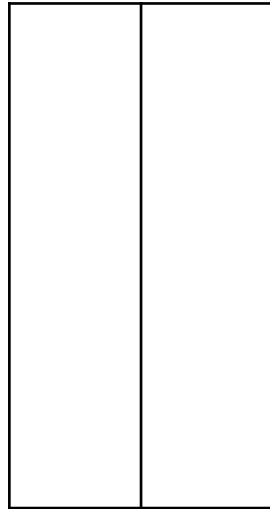
Factor:
Benchmark



Complete factor diagram



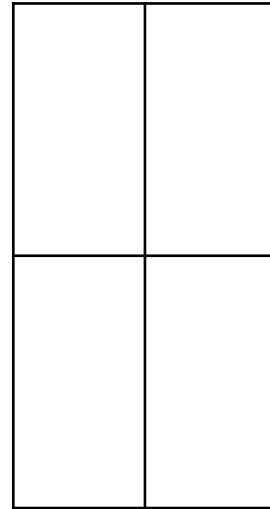
Benchmark



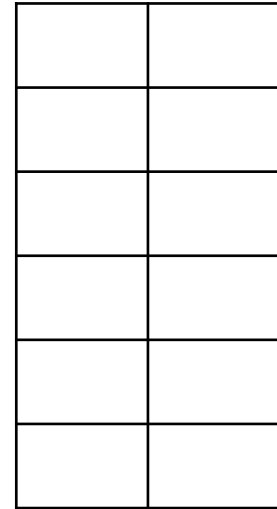
Antibiotics



B12



Interaction

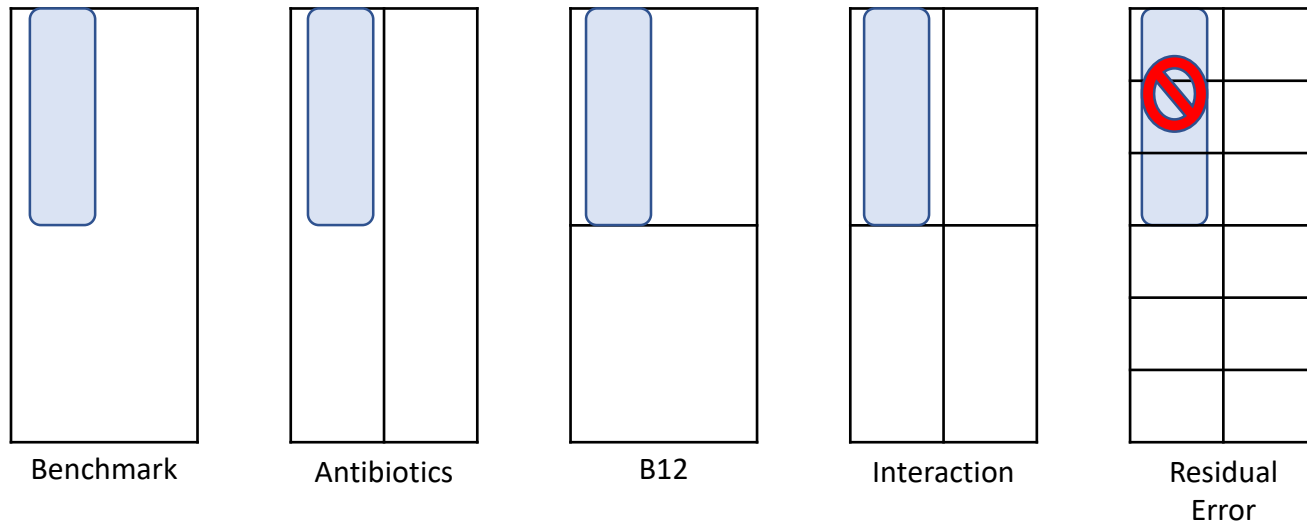


Residual
Error

Finding effects

Effect = average – partial fit

Partial fit = sum of effects for “outside factors”



Outside factors for interaction:

- Benchmark
- Antibiotics
- B12
- ~~Residual Error~~

Averages

		Antibiotics		B12 avgs
		0 mg	40 mg	
B12	0 mg	1.19	1.03	1.11
	5 mg	1.22	1.54	1.38
	Antib. avgs	1.205	1.285	1.245

Data Table

		Antibiotics	
		0 mg	40 mg
B12	0 mg	1.30	1.05
		1.19	1.00
		1.08	1.04
	5 mg	1.26	1.52
		1.21	1.56
		1.19	1.54

Calculation of Effects for upper left observation (1.30)

Factor	Outside Factors	Average	Partial fit	Effect
Benchmark	None	1.245	0	1.245
Antibiotics	Benchmark	1.205	1.245	-.04
B12	Benchmark	1.11	1.245	-.135
Interaction	Benchmark, Antib., B12	1.19	$1.245 + (-.04) + (-.135) = 1.07$.12
Residual Error	Benchmark, Antib., B12, Interaction	1.30	$1.245 + (-.04) + (-.135) + .12 = 1.19$.11

$$1.30 = 1.245 + (-.04) + (-.135) + .12 + .11$$

$$\text{Response} = \text{Benchmark} + \text{Antibiotics} + \text{B12} + \text{Interaction} + \text{Residual Error}$$

Decomposition

1.30	1.05
1.19	1.00
1.08	1.04
1.26	1.52
1.21	1.56
1.21	1.54

Observed
Responses

=

1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245

Benchmark

+

-.04	.04
-.04	.04
-.04	.04
-.04	.04
-.04	.04
-.04	.04

Antibiotics

+

-.135	-.135
-.135	-.135
-.135	-.135
.135	.135
.135	.135
.135	.135

B12

+

.12	-.12
.12	-.12
.12	-.12
-.12	.12
-.12	.12
-.12	.12

Interaction

+

.11	.02
.00	-.03
-.11	.01
.04	-.02
-.01	.02
-.03	.00

Residual
Error

Degrees of freedom

1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245
1.245	1.245

Benchmark

-.04	.04
-.04	.04
-.04	.04
-.04	.04
-.04	.04
-.04	.04

Antibiotics

-.135	-.135
-.135	-.135
-.135	-.135
.135	.135
.135	.135
.135	.135

B12

.12	-.12
.12	-.12
.12	-.12
-.12	.12
-.12	.12
-.12	.12

Interaction

.11	.02
.00	-.03
-.11	.01
.04	-.02
-.01	.02
-.03	.00

Residual Error

df

1

1	R
R	R
R	R
R	R
R	R
R	R

1

1	+
R	R
R	R
R	R
R	R
R	R

1

1	R
R	R
R	R
+	R
R	R
R	R

1

1	+
R	R
R	R
+	+
R	R
R	R

8

1	3
2	4
+	+
5	7
6	8
+	+

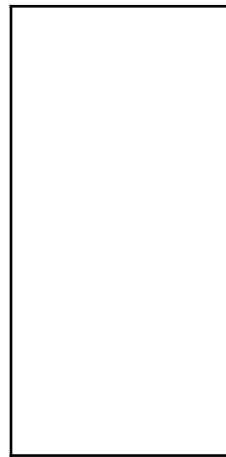
Why not free?

R: repeated

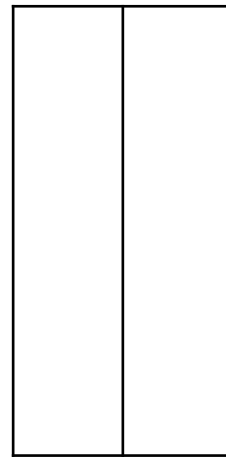
+: sum to zero

Another rule for df

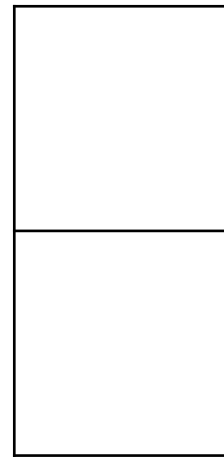
$$DF = (\# \text{ levels}) - (\text{sum of df for outside factors})$$



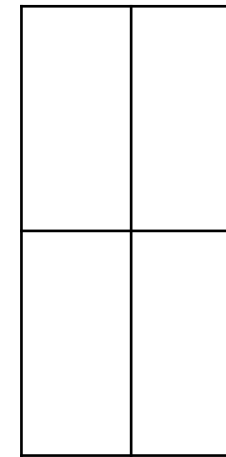
Benchmark



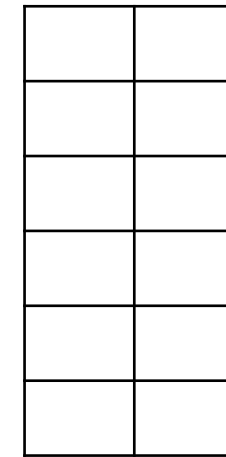
Antibiotics



B12



Interaction



Residual
Error

Factor	Outside Factors	# Levels	Sum of df for outside factors	DF
Benchmark	None	1	0	1
Antibiotics	Benchmark	2	1	1
B12	Benchmark	2	1	1
Interaction	Benchmark, Antib., B12	4	$1 + 1 + 1 = 3$	1
Residual Error	Benchmark, Antib., B12, Interaction	12	4	8