POGIL-style Activities
Introducing Students to Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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eCOTS

Outline

- Motivation
- Activities
  - Developing Bayes' Theorem
  - Applying Bayes' Theorem
- Teaching Environment and Feedback
- Wrap-Up

Motivation

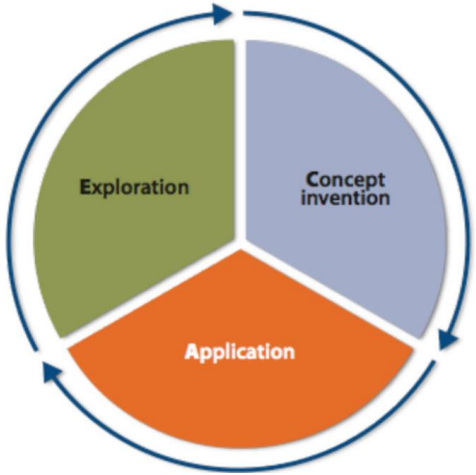
- Relative dearth of Bayesian activities for classroom use
- Attendance (with coauthors) to 2023 Bayes-BATS workshop
  - Tier 1: attend weeklong instructor workshop
  - Tier 2: develop Bayesian teaching and learning materials
  - Tier 3: share resources with wider community

Activities

POGIL: Process Oriented Guided Inquiry Learning

Process Oriented (Cooperative Learning):
Develop Key Process Skills

Guided Inquiry (Constructivism):
Learning Cycle Activities



https://www.pogil.org/

Activity 1

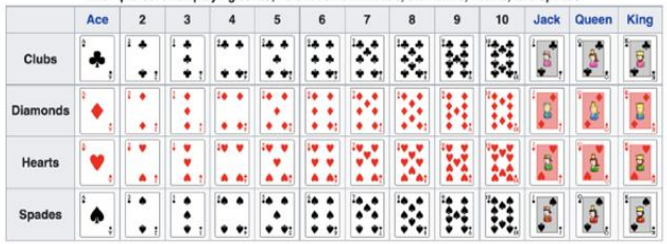
- Title: Developing Bayes' Theorem
- Assumed knowledge: high school algebra, introduction to probability helpful but not necessary

- Description

Activity 1: Developing Bayes' Theorem

Model 1 Review probability using a cards example

Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades



Questions (10 min) **Start time:**

1. Based on the screenshot above:
   a) How many cards are there in a standard deck of cards?
   b) How many red cards are there?
   c) Is the number of black cards the same as the number of red?
   d) How many cards are in the suit hearts?
   e) Are there the same number of cards in the suit clubs?
   f) When you add up the number of hearts, clubs, diamonds, and spades, what number do you get?
   g) Is this the same as the total number of red + black cards?

Activity 1: Developing Bayes' Theorem

Model 2 Using conditional probabilities

The ELISA test for HIV was developed in the mid-1980s during the peak of the AIDS epidemic in the United States. Blood samples were tested to detect whether or not an HIV-infection was present. As with any medical diagnostic test, the results will sometimes be wrong. That is, sometimes a person will actually have an HIV infection and the ELISA test will return a negative result, and other times a person will NOT have an HIV infection but the ELISA test will return a positive result. In the early stages of ELISA's development, the test was found to return correct results more than 90% of the time both when a person was actually HIV-infected and when they were NOT HIV infected.

6. Thought question: Make a prediction for the percentage of blood samples with positive test results that are actually infected with HIV. In other words, if you got a positive test result, what's the probability you are actually infected with HIV?

The table below gives the results of the ELISA test for a hypothetical population of 1 million people. The numbers are based on real data about the effectiveness of the test found in this 1987 article: https://projecteuclid.org/journals/statistical-science/volume-2/issue-3/The-Statistical-Precision-of-Medical-Screening-Procedures–Application-to/10.1214/ss/1177013215.full.

	Positive Test Result	Negative Test Result	Total
Actually HIV infected	4885	115	5000
Actually NOT HIV infected	73630	921370	995,000
Total	78515	921485	1,000,000

Ask Good Questions: #10 My Favorite Theorem (Rossman)

Activity 1: Developing Bayes' Theorem

Model 3 Discovering Bayes Theorem

Let's consider a different infectious disease that is more prevalent than HIV: influenza A (the common flu). We'll assume the ***prevalence*** is 8%. The flu is usually diagnosed via Rapid Influenza Diagnostic Tests (RIDTs). We'll assume these tests have a 92.6% sensitivity and 97.7% specificity just like the ELISA HIV test; these rates are on par with what the FDA requires for RIDTs (https://www.cdc.gov/flu/professionals/diagnosis/rapidlab.htm). Often, this is the only information we have in real life - we don't observe all the counts or results for everyone in the population, but we do have estimates of the prevalence, sensitivity, and specificity.

	Positive Test Result	Negative Test Result	Total
Actually Flu infected			
Actually NOT Flu infected			
Total			1,000,000

f) Using the notation from part e and your results from part d, re-write P(Flu | +) using notation only.

$$P(Flu|+) = \frac{(\quad)(\quad)}{(\quad)(\quad)+(\quad)(\quad)}$$

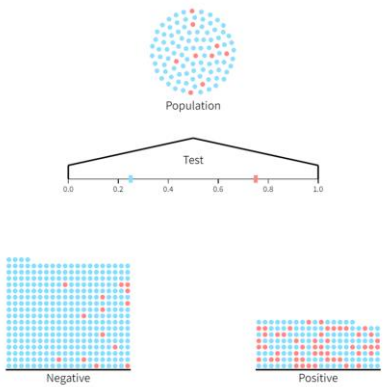
Activity 2

- Title: Applying Bayes' Theorem
- Assumed knowledge: high school algebra, previous exposure to Bayes' Theorem

- Description

Activity 2: Applying Bayes' Theorem

Model 1:



[Seeing Theory](#) (Kunin)

Model 2:

Steve is very shy and withdrawn, invariably helpful but with very little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail.

Which of the following do you find more likely?
- Steve is a librarian
- Steve is a farmer

[3Blue1Brown](#) (Sanderson and Pullen)

Model 3: Sports Predictions

Your soccer team last year:
- Won 25 of 34 games.
- Altogether, played in 13 games with a 1-goal deficit at some point during the game. Of these, won 5, lost 7, and tied 1.

Teaching Environment and Feedback

Institutions

- Azusa Pacific, Math, Physics, & Statistics, R2 private university, HSI & AANAPISI
- Wisconsin Lutheran, Biology, private liberal arts university

* 80 minute class period

Fall 2023 Pilot Runs of Activity 1

Course Name	Type of Course	Number of Students	Incorporation into Classroom	Assessment
Math in Society	General Education	7	stand-alone activity	none
Elementary Statistics	General Education	30	stand-alone activity	feedback form for extra credit
Biostatistics	STEM Elective	25	stand-alone activity	feedback form for extra credit

Spring 2024 Pilot Runs of Activity 1 and 2

Course Name	Type of Course	Number of Students	Incorporation into Classroom	Assessment
Elementary Statistics	General Education	29	stand-alone activity	feedback form for extra credit
Statistical Models	STEM Elective	5	stand-alone activity	activity debrief

Wrap-Up

- Fun for students and instructors
- Length needs adjusting

- PDFs of activities available on eCOTS page or via QR code below
- Contact me with questions! Angela.Ebeling@wlc.edu or 414.443.8644

- Thank you! ☺

Scan me!
Link to pdfs of activities (student and instructor versions), as well as additional info including summaries of student feedback.